The Ohio State University

Modeling the Stellar “Dead Zone”

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Abstract

This study investigates the magnetospheric structure of stellar winds through numerical simulations using the `Athena++` magnetohydrodynamics framework. By adapting existing code to incorporate an isothermal equation of state, we modeled the interaction between a rotating star’s magnetic field and its plasma outflow to determine the resulting magnetic topology. We specifically calculated the dimensionless radius of the last closed field line ($\bar{r}/R$) across a range of stellar spin periods through numerical interpolation methods. Our results demonstrate close alignment with the classical analytical predictions of Mestel & Spruit, with a maximum deviation of less than 7%. These minor variations are attributed to numerical diffusion and boundary instabilities, validating the modified code’s reliability in capturing the “dead zone” of closed magnetic loops. This work establishes a robust baseline for future study, including the refinement of divergence-free interpolation, expansion to three-dimensional geometries, and the integration of Paczynski-Wiita potentials to simulate relativistic effects in compact objects.

Introduction

The evolution of stellar rotation is primarily goverened by the interaction between a star’s magnetic field and its plasma outflow [1]. This interaction creates a **magnetospheric “dead zone”**, a region of closed magnetic field lines where plasma is trapped, effectively increasing the star’s moment of inertia and modulating angular momentum loss [2]. Understanding the geometry of this region is critical for predicting the long-term spin-down of stars and compact objects.

Early analytical work by **Mestel & Spruit (1987)** established that the radius of the last closed field line ($\bar{r}$) results from the competition between magnetic pressure and centrifugal, gravitational, and thermal pressure forces [2]. While foundational, these models often rely on simplifying steady-state assumptions. Recent numerical studies using the `Athena++` framework [1] have extended this work, demonstrating high reliability in modeling magnetized isothermal winds across diverse rotation rates. Such simulations are vital for understanding the rotational evolution of young, highly magnetic neutron stars, where slight variations in ”dead zone” topology significantly impact observable spin-down rates [3].

This project aims to validate a modified isothermal setup within `Athena++` by calculating $\bar{r}/R$ across varying spin periods. By comparing our numerical results to classical predictions of Mestel & Spruit, we establish a baseline for future 3D simulations and relativistic approximations in the study of rapidly rotating magnetars.

Computational Framework

We performed numerical simulations using `Athena++`, an open-source astrophysics MHD code [4]. The framework utilizes a curvilinear coordinate system and algorithms to maintain the divergence-free constraint of the magnetic field ($\nabla \cdot \mathbf{B} = 0$) [4]. For this study, the code was configured in 2D axisymmetry to model the stellar magnetosphere efficiently while preserving the essential physics of the rotating outflow.

We utilized the built-in **isothermal equation of state**, where the pressure P is directly proportional to the density ρ via $P = \rho c_T^2$, where c_T is the constant isothermal sound speed. This simplification allows for a direct comparison with the analytical results by Mestel & Spruit [2], isolating the effects of magnetic torque and stellar rotation from complex thermodynamic cooling and heating gradients [1].

We quantify our rotation speed for comparison through the **dimensionless parameter** ξ_Ω [1],

$$\xi_\Omega = \left.\frac{v_\phi}{v_{\rm esc}}\right|_R = \left(\frac{\Omega^2 R^3}{2GM}\right)^{1/2}. \qquad (1)$$

Magnetosphere Morphology

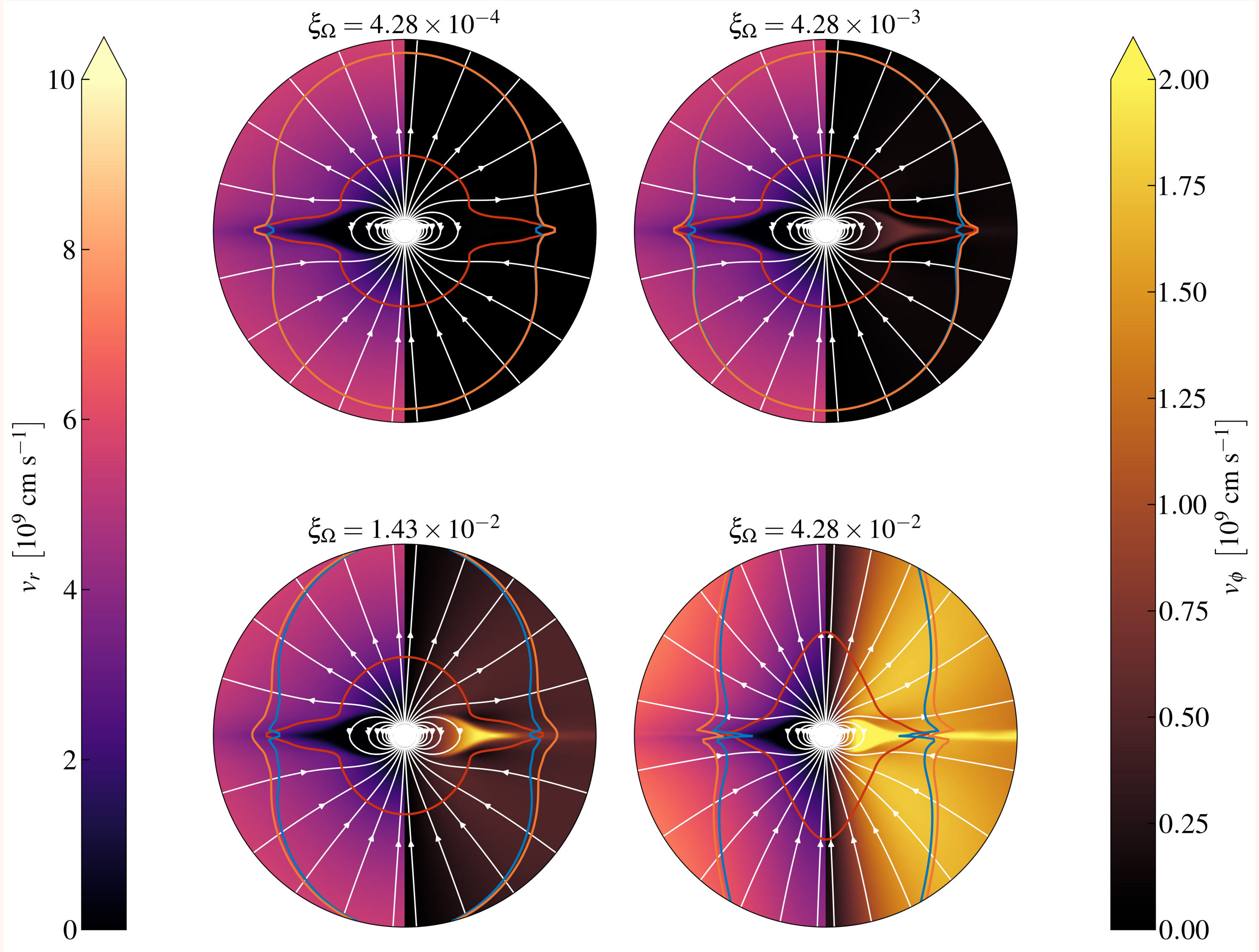

Figure 1. MHD solutions from `Athena++` for four rotation rates characterized by ξ_Ω. The left hemisphere of each plot shows radial velocity v_r while the right shows azimuthal velocity v_ϕ. White streamlines track the magnetic field topology. Critical MHD surfaces are highlighted: the Mach surface (red), Alfvén surface (blue), and Fast Magnetosonic surface (orange). Note the expansion of the closed-field region/the distortion of the Alfvén surface as rotation increases.

Field Line Extraction

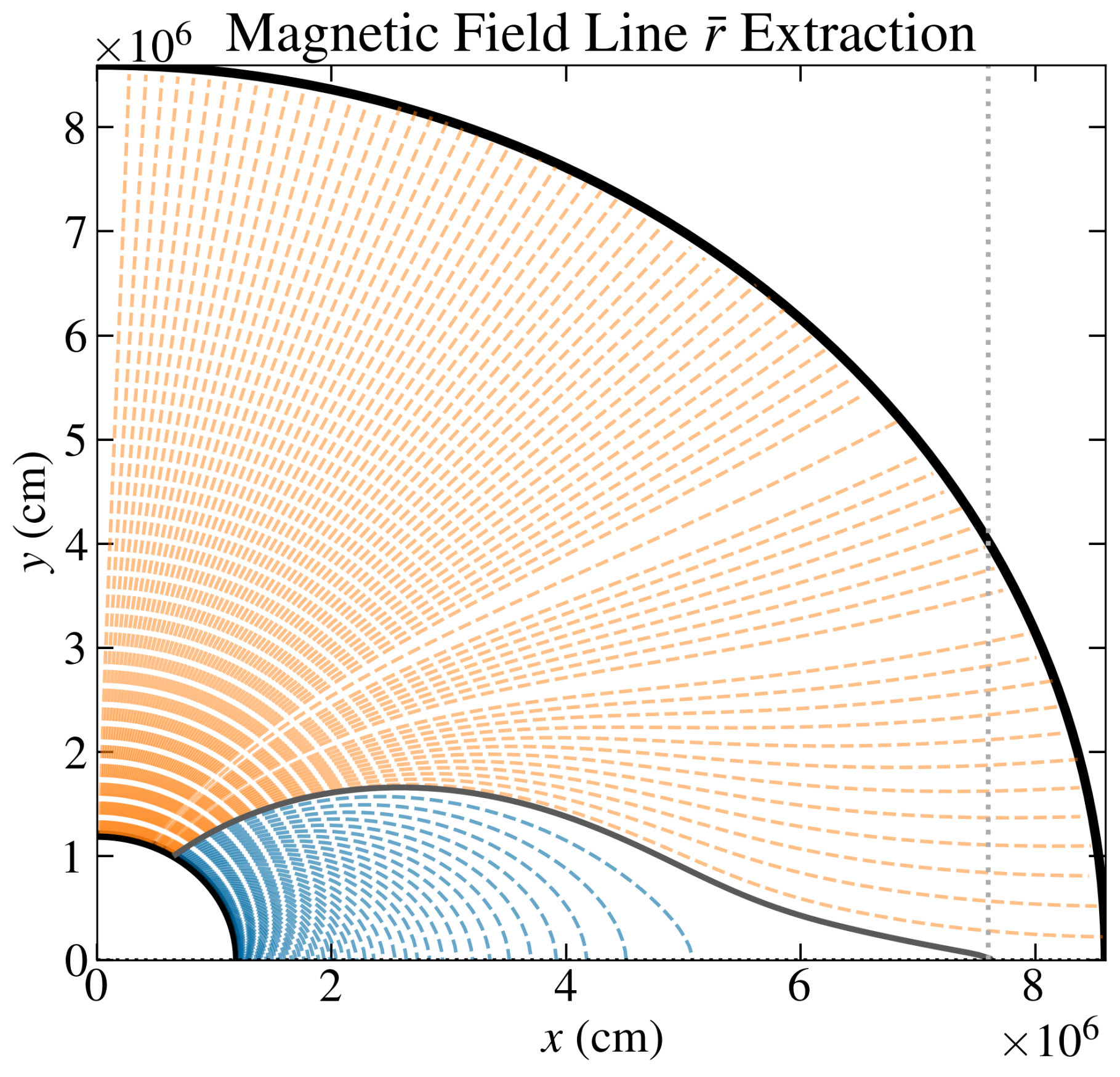

Figure 2. Example numerical extraction of last closed field line $\bar{r}$. Blue dashed lines indicated closed loops within the magnetospheric dead zone, while orange dashed lines represent open field lines integrated into the stellar wind. The solid gray line denotes the extracted $\bar{r}$, with its equatorial crossing defining the Y-point radius. The vertical gray dotted line indicates the prediction for this specific rotation rate, demonstrating the accuracy of the numerical analysis.

In Fig. 2, we find the prediction utilizing the closed form from Mestel & Spruit, utilizing Newton-Raphson inversion to solve for $\bar{r}/R$ [2]:

$$\left(\frac{R}{\bar{r}}\right)^6 = \frac{8\pi(\varrho_0)_{\rm d} c_T^2}{B_0^2} \exp\left[-\frac{GM}{Rc_T^2}\left(1 - \frac{R}{\bar{r}}\right)\right] \exp\left[\frac{1}{2}\frac{\Omega^2 R^2}{c_T^2}\left(\frac{\bar{r}^2}{R^2} - \frac{R}{\bar{r}}\right)\right]. \qquad (2)$$

Validation and Scaling

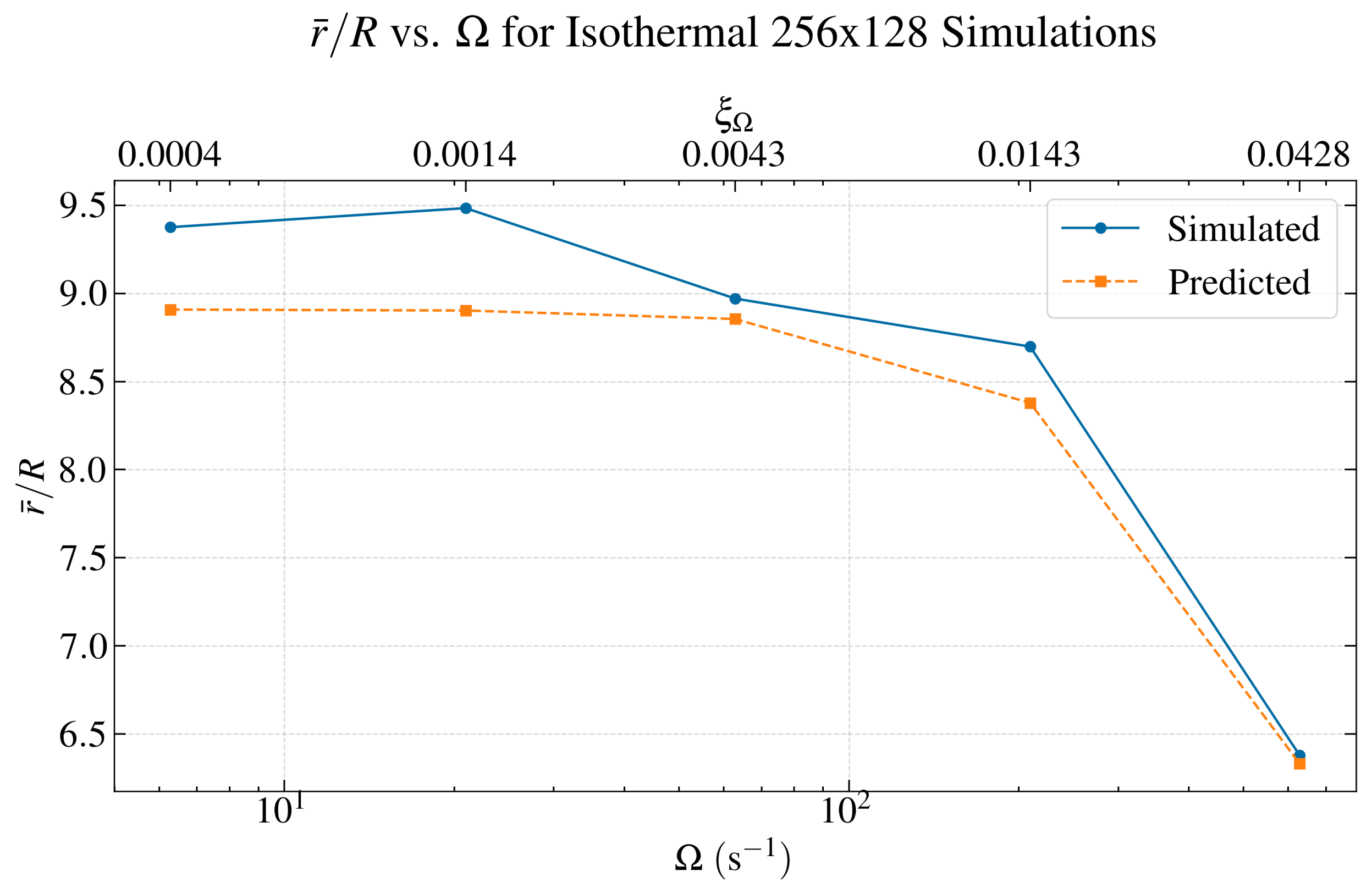

Figure 3. Comparison of the dimensionless Y-point radius $\bar{r}/R$ as a function of stellar angular velocity Ω. Blue circles represent data extracted from the simulations, while orange squares denote theoretical values from Eq. 2 [2]. Close agreement across three orders of magnitude in rotation validates the framework’s ability to model angular momentum loss in diverse stellar environments.

Discussion

- **Regime Recovery:** Simulations successfully capture the three-stage evolution. At low rotation ($\Omega < 10$ s^{-1}), magnetic pressure maintains a stable dead zone. In the “rapid” regime ($\Omega > 10^2$ s^{-1}), centrifugal stretching “snaps” field lines open, driving the observed $\bar{r}/R$ collapse.
- **Model Deviations & Resolution:** A systematic $< 7\%$ overestimation of $\bar{r}$ vs. theory highlights the impact of B_ϕ and numerical diffusion. We expect improvement with increased **angular resolution**. Unlike 1D models, 2D `Athena++` captures the **closed zone**, allowing us to resolve the competition between B-field strength, mass loss ($\dot{M}$), and torque. Higher B expands the closed zone, suppressing $\dot{M}$ while increasing R_A to enhance spin-down.
- **Magnetar Implications:** Validates isothermal `Athena++` as a reliable “litmus test” for relativistic models [1, 3]. Because braking torque depends non-linearly on $\bar{r}$, even **relatively small shifts** in $\bar{r}/R$ significantly alter the fraction of open field lines, driving the rapid spin-down observed in young magnetars.

Future Work: We aim to break axisymmetry with 3D geometries, explore relativistic effects via the Paczyński-Wiita potential, and refine Y-point extraction using higher-order, divergence-free interpolation.

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References

[1] M. J. Raives, M. S. B. Coleman, and T. A. Thompson, “Magnetized rotating isothermal winds,” *Monthly Notices of the Royal Astronomical Society*, vol. 526, pp. 4418–4434, 10 2023.

[2] L. Mestel and H. C. Spruit, “On magnetic braking of late-type stars,” *Monthly Notices of the Royal Astronomical Society*, vol. 226, pp. 57–66, may 1987.

[3] T. Prasanna, M. S. B. Coleman, M. J. Raives, and T. A. Thompson, “The early evolution of magnetar rotation – i. slowly rotating ‘normal’ magnetars,” *Monthly Notices of the Royal Astronomical Society*, vol. 517, pp. 3008–3023, 09 2022.

[4] J. M. Stone, K. Tomida, C. J. White, and K. G. Felker, “The athena++ adaptive mesh refinement framework: Design and magnetohydrodynamic solvers,” *The Astrophysical Journal Supplement Series*, vol. 249, p. 4, June 2020.

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