Order of Magnitude – Group α

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1 A Positive Disaster

Imagine an alien race has come to our solar system with an "electron capture" magnet. They position the magnet at the moon and start ripping away electrons from the moon, making it positively charged, and they conveniently don't repel each other or attract other positive charges when doing so. As the moon continues to get more positively charged, it will eventually "explode" (become unbound).

(a) Calculate the number of electrons you have to remove before the moon explodes/becomes unbound.

We start with our main statement, the conservation of energy that the electrostatic potential energy equals the gravitational potential energy:

$$\mathrm{PE}_{\mathrm{ES}} = \mathrm{PE}_{\mathrm{g}} \tag{1}$$

1.1 The Electrostatic Potential Energy

We use the main form for the electrostatic potential energy. This assumes we are treating the net positive charge as a uniform distribution throughout the volume. We are also ignoring basic structural integrity, treating the moon as a collection of particles where the only binding force is gravity. This ignores details like the moon's tensile strength.

$$\mathrm{PE}_{\mathrm{ES}} = \frac{Q_{\mathrm{enc}}^2}{4\pi\varepsilon_0 r}, \tag{2}$$

where Q_{enc} is the enclosed charge within a Gaussian surface surrounding the moon, r is the radius (of the moon), and ε_0 is the permittivity of free space. If we have n electrons and take the charge of the electron q_e, we can rewrite this as

$$\mathrm{PE}_{\mathrm{ES}} = \frac{(q_e n)^2}{4\pi\varepsilon_0 r}. \tag{3}$$

1.2 The Gravitational Potential and Rearranging

We know that the gravitational potential is given by

$$\mathrm{PE}_{\mathrm{g}} \sim \frac{GM^2}{r}. \tag{4}$$

This gives us the full relation

$$\frac{(q_e n)^2}{4\pi\varepsilon_0 r} = \frac{GM^2}{r}. \tag{5}$$

We want to know how many electrons, n. So, we can rearrange to get

$$n^2 = \frac{4\pi\varepsilon_0 GM^2}{q_e^2} \Rightarrow n = \left(\frac{4\pi\varepsilon_0 GM^2}{q_e^2}\right)^{1/2}. \tag{6}$$

We can see that the radius r actually cancels out, so the only factor that matters here is the mass M. Using the constants in Section 2, we can plug in and solve:

$$n = \left[\frac{4\pi(8.85\times10^{-18}\mathrm{C^2g^{-1}cm^{-3}s^2})(6.67\times10^{-8}\mathrm{cm^3g^{-1}s^{-2}})(7\times10^{25}\mathrm{g})^2}{(1.6\times10^{-19}\mathrm{C})^2}\right]^{1/2} = 1.192\times10^{33}\text{ electrons.}$$

So,

$$\boxed{n \approx 10^{33}\text{ electrons.}}$$

(b) Is this a sensible amount of electrons?

We assume that the moon is of uniform density. Then, the total number of nucleons in the moon is roughly

$$N_{\mathrm{tot}} \sim \frac{3}{2}\frac{M}{m_p}, \tag{7}$$

with the 3/2 coming from the fact that we have 3 different particles but only the mass from 2 of them are non-negligible (the proton and neutron).

Utilizing our known values for M and m_p, we can find the total number of nucleons as

$$N_{\mathrm{tot}} \approx \frac{3}{2}\frac{7\times10^{25}\mathrm{g}}{1.67\times10^{-24}\mathrm{g}} \approx 6.29\times10^{49}\text{ nucleons.}$$

Hence,

$$\boxed{N_{\mathrm{tot}} \approx 10^{49}\text{ nucleons.}}$$

So, this is a sensible result in the sense that we don't have to strip away more electrons than there are total nucleons in the moon itself. It is also physically reasonable that we don't need to strip the moon bare, as gravity is incredibly weak compared to the electrostatic force. Because of this disparity, even a microscopic imbalance in charge (with respect to the total number of nucleons) creates enough repulsive force to overcome the entire gravitational pull of the moon's mass.

From an engineering standpoint, however, this "electron capture magnet" is likely impossible. The energy required to remove that last electron against an already massive positive charge would likely be astronomical, and it is unlikely you could even reach that point.

2 Constants for the Problem

Mass of the Moon M

$$M \approx 7\times10^{22}\text{ kg} = 7\times10^{25}\text{ g.}$$

Permittivity of Free Space ε_0

$$\varepsilon_0 \approx 8.85\times10^{-12}\text{ C}^2\text{ kg}^{-1}\text{ m}^{-3}\text{ s}^2 = 8.85\times10^{-18}\text{ C}^2\text{ g}^{-1}\text{ cm}^{-3}\text{ s}^2.$$

Electron Charge q_e

$$q_e \approx 1.6\times10^{-19}\text{ C.}$$

Mass of a Proton m_p

$$m_p \approx 1.67\times10^{-27}\text{ kg} = 1.67\times10^{-24}\text{ g.}$$

Gravitational Constant G

$$G \approx 6.67\times10^{-11}\text{ m}^3\text{ kg}^{-1}\text{ s}^{-2} = 6.67\times10^{-8}\text{ cm}^3\text{ g}^{-1}\text{ s}^{-2}.$$